Available online at www.sciencedirect.com
SCIENCE DIRECT®
Computer Networks 49 (2005) 580–592
Computer Networks
www.elsevier.com/locate/comnet

A nonlinear control theoretic analysis to TCP–RED system

Fengyuan Ren [a,*], Chuang Lin [a], Bo Wei [b]

[a] Network Research Institute, Department of Computer Science and Technology, Tsinghua University, Beijing 100084, PR China
[b] Department of Electrical Engineering, Southern Methodist University, Dallas, TX 75275, USA

Received 16 June 2004; received in revised form 27 December 2004; accepted 10 January 2005
Available online 21 March 2005

Responsible Editor: G. Morabito

Abstract

Random early detection (RED) is an effective congestion control mechanism acting on the intermediate node. The considerable recent studies have investigated on the stability of TCP/RED system. In this paper, we firstly summarize the contributions and limitations included in the existing works. To reveal a more comprehensive reason why TCP/RED system is apt to oscillate, a new analysis framework is constructed to sufficiently merge the existing valuable results with the aid of the describing function approach, which is rather mature in nonlinear control theory. After a brief introduction of the describing function approach, a proposition about TCP/RED system stability criterion is proposed. Subsequently, we use this criterion to quantitatively analyzed why gentle-RED is more stable than RED, and to investigate the impact of typical system parameters, such as propagation delay, load level, link capacity, and averaging weight factor, on TCP/RED system stability in detail. The simulation results validate our analysis.

Keywords: Stability; Limit cycle; Describing function; Active queue management

1. Introduction

Since Nagle [1] reported on the congestion collapse due to TCP connections unnecessarily retransmitting packets in 1984, the congestion control in IP networks has been a recurring problem. The end-to-end flow control mechanism invented by Jacobson forms the basis for TCP congestion control, and becomes mandatory requirements for all Internet hosts. Few would argue that the traditional TCP congestion control mechanism have served the Internet remarkably well and formed the basis for its survival and success, however, it is not sufficient to provide perfect performance in

* Corresponding author. Tel.: +86 10 62783596; fax: +86 10 62771138.
E-mail addresses: renfy@csnet1.cs.tsinghua.edu.cn (F. Ren), clin@csnet1.cs.tsinghua.edu.cn (C. Lin), bwei@engr.smu.edu (B. Wei).

1389-1286/$ - see front matter
doi:10.1016/j.comnet.2005.01.016

all circumstances, especially with the rapid growth in scale of network and the strong requirements to QoS support, because the corresponding algorithms merely focus on functions at the end-host, but the control which can be accomplished at end systems is limited. Accordingly, it is spontaneous to explore how to utilize the intermediate node ability to enhance the end-to-end congestion control. Following this consideration, ECN (explicit congestion notification) [2] and AQM (active queue management) [3] are successively developed. The goals of AQM aim at decreasing the end-to-end delay experienced by flows through maintaining the smaller queue in intermediate nodes, at the same time keeping the higher link utilization through avoiding the queue emptying. RED [4] was originally proposed to achieve fairness among sources with different burst attributes and to control the queue length to the expected range, which just meets the requirements of AQM, so RED algorithm was recommended as the only candidate algorithm for AQM in RFC2309 [3].

The basic idea of RED is to sense the coming congestion and try to inform the senders by either dropping or marking the packets. The dropping probability is updated according to the control law defined by a piecewise function of average queue length. This scheme includes four setting parameters: (1) $p_{\max}$, the maximum packet dropping probability, (2) w_q, the weighted factor, (3) $\min_{\text{th}}$, the minimum threshold of average queue length, (4) $\max_{\text{th}}$ the maximum threshold of average queue length.

RED is rather effective if correctly configured, however, many subsequent studies verified that RED is unstable and too sensitive to parameter configuration [5], and extremely hard to reduce the oscillation by tuning RED parameters [6], even more, some literatures discouraging implementation of RED (e.g. [7]) argued that there was insufficient consensus on how to select the values of the setting parameters because most of the rules for setting these parameters are empirical, and come from practical experience, and that RED could not be modified to provide a drastic improvement in performance. Therefore, some researchers began to explore why RED can induce network instability and major traffic disruption if not properly configured.

TCP/AQM oscillation may be caused by many factors, such as AIMD (additive increase multiplicative decrease) strategy employed by TCP flow control procedure, the noise-like traffic that are not effectively controlled by TCP (e.g. short lived TCP sessions), and non-responsive flows that are scarcely controlled by the sources (e.g. burst UDP traffic). However, more fundamentally, it is due to AQM algorithm itself. Many papers have theoretically analyzed TCP/RED system in framework of feedback control theory based on the continuous-time model (e.g. [8–12] etc.) or discrete-time model (such as [13] and [14]) after having made some necessary simplification and assumption, and finally provided some very revelatory and significant conclusions and judgments. In most of the existed works, the linear control theoretic approaches are frequently used. As for the complex nonlinear network system, they may not be the most appropriate choice, at least still have some limitations. In this study, we will apply the describing function (DF) approach, which is mature in nonlinear control theory, to analyze TCP/RED system, which is a typical nonlinear system since RED algorithm itself includes a structural nonlinear component, i.e. the piecewise control law. In our work, we will sufficiently take the existed results into consideration, and try to synthesize a stability criterion for TCP/RED system to more accurately and comprehensively reveal why TCP/RED is apt to oscillate. The remainder is organized as follows. In Section 2, the limitations in existing works are explained. In Section 3, the DF method and the stability criterion based on it are introduced in brief. In Section 4, a stability condition for TCP/RED system is deduced. Subsequently, we discuss the impact of typical network parameters on the stability, such as propagation delay, load level and link capacity, moreover do some simulations to support our analysis. Finally, a conclusion is drawn in Section 6.

2. Related works and their limitations

2.1. Analysis based on continuous-time model

Misra et al. developed a dynamic model of TCP behavior using fluid-flow and stochastic differential equation analysis. Simulation results demonstrated that this model accurately capture the dynamics of TCP [15]. Subsequently, Hollot et al. used this model as a starting point to perform their analysis [8]. The inherently nonlinear model presented in [15] was converted into a linear system with the technique of local linearization, and RED algorithm was assumed to only work in section between $\min_{th}$ and $\max_{th}$ then the dropping packet probability is proportional to the average queue length, so it is described by $p = Kq$ with $K > 0$ and $p \in [0, 1]$. With these simplification and assumptions, they obtained the linear analysis model depicted in Fig. 1. The meanings of parameters presented in Fig. 1 are as follows:

$$T_q = R, \quad T_t = R^2C/2N, \quad T_a = -1/C\ln(1 - w_q),$$
$$K = (RC)^3/(2N)^2, \quad L_{\text{red}} = \frac{p_{\max}}{\max_{\text{th}} - \min_{\text{th}}}, \tag{1}$$

where R is round-trip time (RTT), N denotes number of TCP sessions, and C stands for link capacity.

Based on this continuous-time model, Hollot et al. provided the following proposition describing stability and robustness of the linear system applying the well-developed tools in classical linear feedback control theory.

As the extension to their research, Hollot et al. directly addressed the nonlinearities appearing in the TCP fluid-flow model and established some stability results in [10]. However, the nonlinearity existing in RED algorithm was also ignored, thus, the results are effective only when the average queue length lies between $\min_{th}$ and $\max_{th}$ The only difference was that the linear model of plant was replaced with the nonlinear one.

In [9], a general nonlinear model of TCP/RED was developed, then after linearizing it around equilibrium and making several simplifying assumptions, the sufficient stability condition for TCP/RED was obtained. The main contribution in [9] was to replace the single-link identical-source model in [8] with a general model with heterogeneous sources. However, just like in [8], RED algorithm itself was also simply approximated as a linear component. Under this assumption, some possible factors leading to the queue oscillation have been successfully identified, however, the others will be likely hidden. The following two facts can support this supposition.

Case 1. The propositions presented in the previous works cannot definitely explain why the oscillations occurring in TCP/RED system always keep the nearly same amplitudes, namely they seem to be limit cycle.

Although the previous propositions were obtained according to the frequency domain criterion, we still may illustrate them in the complex plane because the Nyquist stability criterion in frequency domain is equivalent with the corresponding one in complex domain. It is well known that the stability of linear feedback system depends on the distribution of the roots of the closed-loop characteristic equation [17]. As shown in Fig. 2, when all roots stay in LHP (left half plane), the closed-loop system is absolutely stable, whereas, once there is a root in RHP (right half plane), the linear feedback system will diverge. If the root just lies on the imaginary axis, the system likely oscillates with the same amplitude, unfortunately, it is only a critical state, and hard to be held, so it is impossible to occur the oscillation like a limit cycle under noise disturbance. However, actually

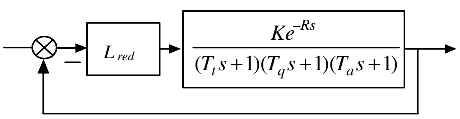

Fig. 1. The linear model of TCP/RED.

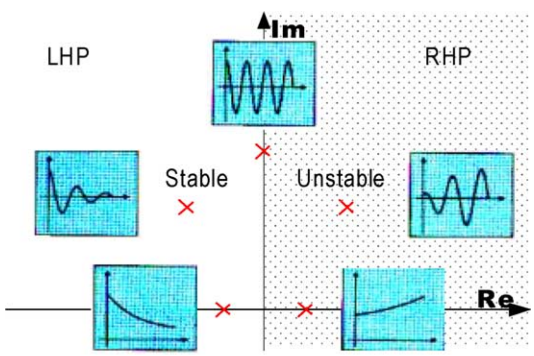

Fig. 2. Stability criterion in the complex domain.

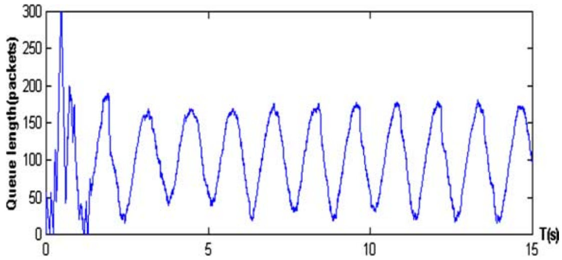

Fig. 3. Typical unstable queue controlled by RED.

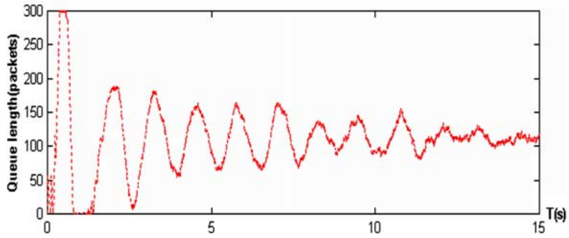

Fig. 5. Queue controlled by gentle-RED.

the numerous simulations have verified that the unstable router queue always evolves as shown in Fig. 3 [8,9,13,14], and never leads to buffer overflow because the queue diverges. In other words, it is not completely reliable to find the reason why TCP/RED system is readily unstable only applying the Nyquist stability criterion in linear control theory.

Case 2. The propositions presented in previous works cannot prove that gentle-RED [16] is more stable than RED. However, the plentiful experiments have verified this fact.

In almost all works based on the continuous-time model, RED algorithm was approximated as a linear amplifying component whose gain is L_{red} as shown in Fig. 1. If the gentle-RED algorithm is analyzed using the same approach, the stability region of gentle-RED should be identical with that of RED, because gentle-RED works like RED in section between $\min_{th}$ and $\max_{th}$ as shown in Fig. 4. Whereas, observing the experiment results described in [16], it is practically obvious that gentle-RED is indeed more stable and robust than RED. We repeat the experiment in Fig. 3 with the same network configuration and parameter setting except for replacing RED with gentle-RED. The queue evolution is plotted in Fig. 5. Obviously, it is asymptotically stable, but the existing propositions have no way to explain this phenomenon.

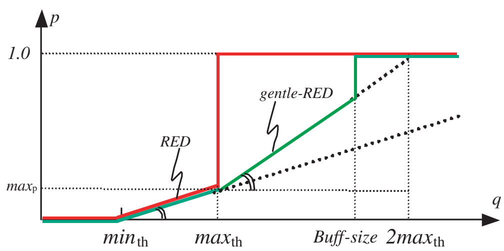

Fig. 4. RED and gentle-RED.

2.2. Analysis based on discrete-time model

Some researchers argued that the continuous-time model was incapable of analyzing the TCP/RED system, so they followed another way. Firoiu and Borden constructed a model of average queue length when TCP flows pass through a queue system with fixed drop probability, then combined it with RED's control rule, and derived the steady state behavior of the resulting feedback control system, finally gave a qualitative illustration about why RED is apt to oscillate and some recommendations for configuring RED control function [13]. At a word, the discrete time model developed in [13] maps the average queue length as a function of dropping packet probability $\boldsymbol{p}$ and the average throughput per flow $\boldsymbol{\pi}$, i.e. $\boldsymbol{\pi} = \boldsymbol{C}/\boldsymbol{N}$ ($\boldsymbol{C}$ is capacity of link and $\boldsymbol{N}$ is number of connections).

Using the schematic method in [13], we can explain why gentle-RED is more stable than RED under some network environment. In Fig. 6, $q_1(p)$ and $q_2(p)$ stand for the two different "queue laws", under different network parameters configuration. Since the equilibrium point **B** is convergent, both RED and gentle-RED are stable when they work in the network defined by $q_2(p)$. On the contrary, when the equilibrium point is suited at **A** point, the TCP/RED system, although attracted by this point, cannot stay there, since the value of p is not determined. However, C point is still a convergent equilibrium point since the

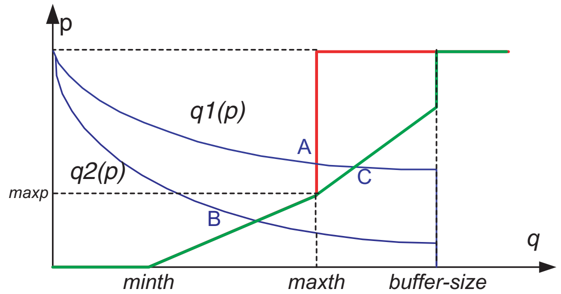

Fig. 6. Gentle-RED/RED operating point converges or diverges.

gentle-RED algorithm extends the linear segment in "control law". Obviously, this analysis emphasizes the impact of the structural nonlinear component in "control law" on the stability.

Priya Ranjan continued Firoiu and Borden's work [14]. After modifying the model presented in [13] with simpler TCP throughput function, they showed that the simplified model exhibits a rich variety of bifurcation behavior leading to chaotic behavior of the computer network. The bifurcations occur as control parameters are slowly varied, moving the dynamics from a stable fixed point to oscillatory behavior, and finally to a chaotic state. In these two studies, the researchers paid more attention to the RED algorithm itself, and insisted that the characteristics of the RED control law, such as its structural nonlinearity and the slope of the segment between $\min_{th}$ and $\max_{th}$ might influence the stability of the feedback system and/or its rate of convergence to the equilibrium point. These arguments are insightful, but there are also some limitations. On one hand, the investigations were qualitative, and did not provide the quantitative conclusion like those in [8,9], on the other hand, most of works omitted the impact of TCP control subsystem, such as link capacity and the number of connections, especially the delay, which is prominent in deteriorating the network stability.

If the works from above two aspects can be integrated rationally, it is likely to give a more comprehensive conclusion, and to obtain an in-depth understanding why the TCP/RED system readily oscillates. In other words, if the model shown in Fig. 7 is used to analyze the stability of RED algorithm, we may get more. Comparing with Fig. 1, the linear amplifying element $\boldsymbol{L}_{\rm red}$ in Fig. 1 is replaced with the nonlinear element, which more accurately describes the TCP/RED compounded system. This substitution will sufficiently take account of the intrinsic nonlinearity element existing in RED or gentle-RED, thus the final analysis results will likely cover the reasons revealed by [13,14]. This idea motivates our work in this paper.

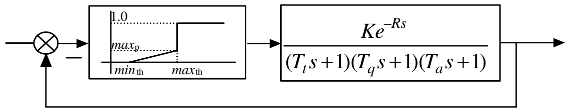

Fig. 7. The nonlinear model of TCP/RED.

In order to analyze the nonlinear system presented in Fig. 7, we need to apply a well-developed approach in nonlinear control theory, i.e. describing function approach. For the convenience of the latter discussion, in next section, it will be briefly introduced.

3. Describing function approach

3.1. Definition of DF

The typical nonlinear control system shown in Fig. 8 is a general form of the system depicted in Fig. 7, it consists of a linear dynamic plant having the transfer function $G(s)$ and preceded by a single, static nonlinear element. The nonlinear characteristic is defined by

$$u = \Phi(e). \tag{2}$$

The output of the nonlinearity may be arbitrary with the input $e = X\sin(\omega t)$, nevertheless, can be represented by a Fourier series:

$$u(t) = U_0 + \sum_{n=1}^{\infty} B_n \sin(n\omega t) + C_n \cos(n\omega t). \tag{3}$$

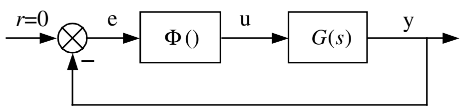

Fig. 8. Typical nonlinear control system.

But the assumed low-passed characteristic of the plant implies that the harmonics of the input beyond the first (i.e. the fundamental) are filtered out by the plant and do not appear in its output. Hence, for the purpose of assessing stability, we can approximate the input to the plant by

$$u(t) \approx B_1 \sin(\omega t) + C_1 \cos(\omega t). \tag{4}$$

Rewriting (4) as

$$u(t) = B_1 \sin \omega t + C_1 \cos(\omega t) = \Theta[N(A)A\mathrm{e}^{\mathrm{j}\omega t}]. \tag{5}$$

Here, Θ denotes the *imaginary part* of the bracketed expression, and

$$N(A) = \Omega(A,\omega) + \mathrm{j}\widehat{\Omega}(A,\omega) = \frac{B_1 + \mathrm{j}C_1}{A}. \tag{6}$$

Eq. (6) is defined as the *describing function* (DF) of the nonlinear component $\Phi()$. It is a complex number, and the real and imaginary parts of the describing function are the coefficients of the fundamental terms in the Fourier series expansion of the output of the nonlinearity, divided by the amplitude A of the input. Thus, according to the theory of Fourier series, the real and imaginary parts of the describing function of the nonlinear function $\Phi()$ are given by

$$\Omega(A) = \frac{1}{\pi A}\int_0^{2\pi} \Phi(A \sin \omega t) \sin(\omega t)\,\mathrm{d}(\omega t), \tag{7}$$

$$\widehat{\Omega}(A) = \frac{1}{\pi A}\int_0^{2\pi} \Phi(A \sin \omega t) \cos(\omega t)\,\mathrm{d}(\omega t). \tag{8}$$

3.2. Stability criterion

With the describing function of nonlinearity, the stability of the nonlinear system can be investigated using the conventional linear techniques with the nonlinearity replaced by an amplitude dependent gain, namely its describing function $N(A)$. As an illustration, the extended Nyquist criterion is explained. Assume that a linear dynamic plant $G(\mathrm{j}w)$ is stable. After the frequency response locus $G(\mathrm{j}w)$ and the negative reciprocal of the describing function $-1/N(A)$, are plotted on a Nyquist Diagram as shown in Fig. 9, the following conclusion is meeting.

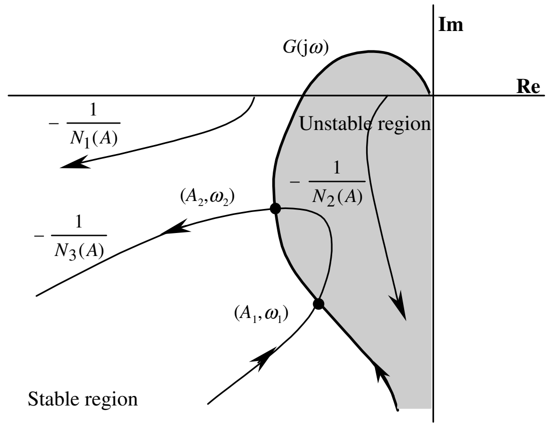

Fig. 9. Illustration of stability.

1. If the negative reciprocal of the describing function of nonlinear component is not surrounded by the linear plant $G(\mathrm{j}w)$ locus, the system is stable, such as $-1/N_1(A)$.
2. If the $-1/N(A)$ locus is surrounded by the $G(\mathrm{j}w)$ locus, the system is unstable, just as $-1/N_2(A)$.
3. When the $-1/N(A)$ locus intersects with the $G(\mathrm{j}w)$ locus, the limit cycle (i.e. self-oscillation) may exist. The corresponding amplitude A and frequency ω are solutions to the characteristic equation (9).

$$1 + N(A)G(\mathrm{j}\omega) = 0. \tag{9}$$

The oscillation may be stable or unstable. Like the $-1/N_3(A)$ shown in Fig. 9, there are two intersections (A_1,ω_1) and (A_2,ω_2), then two limit cycles are possible. The nature of these limit cycles can be assessed through reasoning as follows. If the system were to operate at the intersection (A_1,ω_1), a slight increase in gain would drive the system into the unstable region, causing a still larger oscillation. Hence the intuition would suggest that the limit cycle predicted at (A_1,ω_1) is unstable. On the other hand, if the system were to operate at (A_2,ω_2), a slight increase in gain would drive the system into the stable region. To sustain the limit cycle, the system adjusts itself to recover this operating point. This suggests that the limit cycle (A_2,ω_2) is stable. The details about describing function and stability criterion can be found in [18].

4. Stability criterion of TCP–RED system

In order to analyze the stability of RED (gentle-RED)/TCP system using describing function approach, we firstly need to get the describing function of the nonlinear elements shown in Fig. 4, According to the definition of the describing function (6)–(8), combining with Fig. 10, we obtain the expression (10)

$$N_{\mathrm{RED}}(X) = \frac{2}{\pi X}\int_{\theta_1}^{\pi/2} y_{\mathrm{RED}}(t)\sin\omega t\,\mathrm{d}\omega t = \frac{2}{\pi X}\left\{\int_{\theta_1}^{\theta_2}\frac{\max_p}{H-L}(X\sin\omega t - L)\sin\omega t\,\mathrm{d}\omega t + \int_{\theta_2}^{\pi/2}\sin\omega t\,\mathrm{d}\omega t\right\}, \tag{10}$$

$$N_{0\mathrm{gRED}}(X) = \begin{cases} \dfrac{\max_p}{\pi(1-\gamma)}\left\{\sin^{-1}\beta - \sin^{-1}\beta\gamma + \beta\left[(2\gamma-1)\sqrt{1-\beta^2} - \gamma\sqrt{1-(\beta\gamma)^2}\right]\right\} \\ \quad + \dfrac{2(2\max_p - 1)\beta}{\pi}\sqrt{1-\beta^2} + \dfrac{1-\max_p}{\pi}\left[\dfrac{\pi}{2} - \sin^{-1}\beta + \beta\sqrt{1-\beta^2}\right] & B > X \geqslant H \\ \dfrac{\max_p}{\pi(1-\gamma)}\left\{\sin^{-1}\beta - \sin^{-1}\beta\gamma + \beta\left[(2\gamma-1)\sqrt{1-\beta^2} - \gamma\sqrt{1-(\beta\gamma)^2}\right]\right\} \\ \quad + \dfrac{2\beta(1-2\max_p)}{\pi}\left(\sqrt{1-\alpha^2} - \sqrt{1-\beta^2}\right) + \dfrac{1-\max_p}{\pi}\left(\sin^{-1}\alpha - \sin^{-1}\beta \right. & X \geqslant B \\ \quad \left. -\alpha\sqrt{1-\alpha^2} + \beta\sqrt{1-\beta^2}\right) + \dfrac{2\beta}{\pi}\sqrt{1-\alpha^2}. \end{cases} \tag{14}$$

where

$H = \max_{\mathrm{th}}$,

$L = \min_{\mathrm{th}}$ and B stands for buffer size.

Let

$$\gamma = \frac{L}{H} \quad \beta = \frac{H}{X} \quad \alpha = \frac{B}{X}. \tag{11}$$

Define the relative describing function as follows:

$$N_0(X) = HN(X). \tag{12}$$

Thus, we have the relative describing function of the RED algorithm:

$$N_{0\mathrm{RED}}(X) = \frac{\max_p}{\pi(1-\gamma)}\left\{\sin^{-1}\beta - \sin^{-1}\beta\gamma + \beta\left[(2\gamma-1)\sqrt{1-\beta^2} - \gamma\sqrt{1-(\beta\gamma)^2}\right]\right\} + \frac{2\beta}{\pi}\sqrt{1-\beta^2}. \tag{13}$$

In the same way, we can obtain the relative describing function of the gentle-RED algorithm:

Under the concept of asymptotical stability, we give the following proposition.

Proposition. *If* $G_0(\mathrm{j}\omega_c) > \max_{X\geqslant H}\frac{-1}{N_{0\mathrm{RED}}(X)}$ *or* $G_0(\mathrm{j}\omega_c) > \max_{X\geqslant H}\frac{-1}{N_{0\mathrm{gRED}}(X)}$, *then the queue controlled by RED or gentle-RED is stable where* $G_0(\mathrm{j}\omega) = G(\mathrm{j}\omega)/H$, ω_c *is the cross-frequency of* $G_0(\mathrm{j}\omega)$, *i.e. the root of Eq. (15) at region* $[\frac{\pi}{2T_q}, \frac{3\pi}{2T_q}]$

$$tgT_q\omega + \frac{\omega(T_t + T_q + T_a) - \omega^3 T_t T_q T_a}{1 - \omega^2(T_t T_q + T_t T_a + T_q T_a)} = 0. \tag{15}$$

Proof. Without loss generality, randomly select a network configuration, and assume that both RED and gentle-RED are configured with the com-

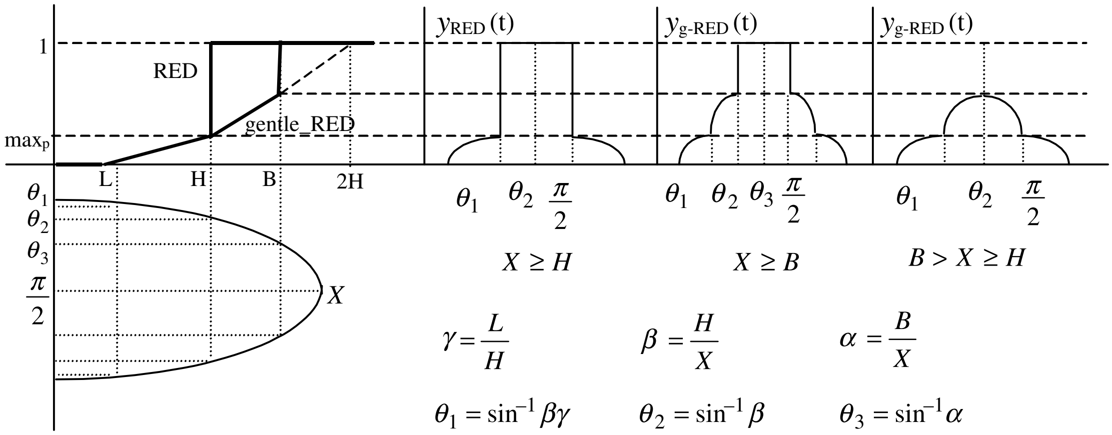

Fig. 10. Describing functions of RED and gentle-RED.

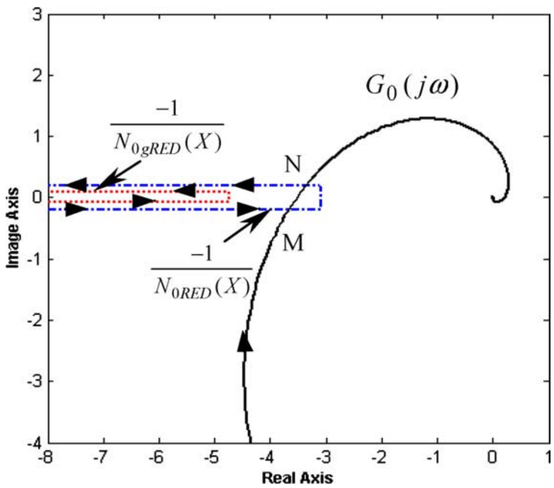

Fig. 11. Nyquist diagram.

mon parameter values. Plot the frequency response locus $G_0(j\omega)$ and the negative reciprocal of the relative describing function $-1/N_0(X)$ on the Nyquist diagram as shown in Fig. 11, both $-1/N_{0red}(X)$ and $-1/N_{0gred}(X)$ lie in the real axis, meanwhile they are convex function of amplitude X, namely when X increases to a special value, $-1/N_{0red}(X)$ (or $-1/N_{0gred}(X)$) will reach its maximum value. If ω_c stands for the cross-frequency of $G_0(j\omega)$, $(G_0(j\omega_c), 0)$ is the intersection between locus $G_0(j\omega)$ and the real axis. Assume that $\max_{X\geqslant H}\frac{-1}{N_{0RED}(X)}$ (or $\max_{X\geqslant H}\frac{-1}{N_{0gRED}(X)}$) denotes that the maximum value of $-1/N_{0red}(X)$ (or $-1/N_{0gred}(X)$). Obviously, if $G_0(j\omega_c) > \max_{X\geqslant H}\frac{-1}{N_{0RED}(X)}$ (or $G_0(j\omega_c) > \max_{X\geqslant H}\frac{-1}{N_{0gRED}(X)}$), the $-1/N_{0red}(X)$ (or $-1/N_{0gred}(X)$) locus must not be surrounded by the $G_0(j\omega)$ locus, the system will be absolute stable according to the previous stability criterion. Since ω_c is the cross-frequency of $G_0(j\omega)$, the phase of $G_0(j\omega)$ is $-\pi$, we have:

$$-arctgT_t\omega_c - acrtgT_q\omega_c - arctgT_a\omega_c - \omega_c R = -\pi.$$

Namely

$$-tg\omega_c R = tg(arctgT_t\omega_c + acrtgT_q\omega_c + acrtgT_a\omega_c).$$

Then

$$tgT_q\omega_c + \frac{\omega_c(T_t+T_q+T_a) - \omega_c^3 T_t T_q T_a}{1-\omega_c^2(T_tT_q+T_tT_a+T_qT_a)} = 0.$$

Thus, ω_c should be the root of Eq. (15) at region $[\pi/2T_q, 3\pi/2T_q]$. □

5. Analysis and discussion

In this section, we will apply the above proposition to explain some phenomena occurring in the router queue, such as, why gentle-RED is more stable that RED, and why the unstable RED queue always oscillates with the nearly same amplitude? Moreover, We will also investigate the impact of the configuration parameters, *including link capacity, delay and load etc.* on the stability of TCP/

RED system *in aid of our criterion*. It is noted that our stability criterion is implicit in system parameters, because $G_0(j\omega_c)$ is a function of parameter R, N and C, and $N_{0RED}(X)$ or $N_{0gRED}(X)$ is a function of parameter $\min_{th}$, $\max_{th}$, w_q and $\max_p$. Therefore, the criterion is incapable to directly provide the conditions with which the system parameters should satisfy, we need some trivial numerical computing in aid of some professional tools, such as Matlab etc.

5.1. Gentle-RED is more stable than RED

The queue evolutions presented in Figs. 3 and 5 are obtained on the popular network simulation platform NS2 [19], and the concrete configuration parameters as follows: $N = 60$, $R = 0.5$ s, $C = 3750$ packets/s, $L = 75$ packets, $H = 150$ packets, $B = 300$ packets, $\max_p = 0.1$ and $w_q = 0.002$. The corresponding the frequency response locus $G_0(j\omega)$ and the negative reciprocal of the relative describing function $-1/N_0(X)$ have been depicted in Fig. 11. Since $G_0(j\omega_c) < \max_{X\geqslant H}\frac{-1}{N_{0RED}(X)}$, but $G_0(j\omega_c) > \max_{X\geqslant H}\frac{-1}{N_{0gRED}(X)}$, the $-1/N_{0gred}(X)$ locus is not surrounded by the $G_0(j\omega)$ locus, while there are two intersections between $-1/N_{0gred}(X)$ and .locus, thus RED is unstable but gentle-RED is asymptotically stable. Generally speaking, $\max_{X\geqslant H}\frac{-1}{N_{0RED}(X)}$ is always somewhat larger than $\max_{X\geqslant H}\frac{-1}{N_{0gRED}(X)}$, in other words, the gentle-RED algorithm is more stable than the RED algorithm.

5.2. Self-oscillation in queue

Although Fig. 11 tells that RED is unstable under this special configuration, the queue impossibly diverges according to the extended Nyquist criterion, because the self-oscillation (i.e. limit cycle) occurring at $\boldsymbol{N}$ point can be proved to be stable. In order to show more enough evidence, we further solve the coordinate of $\boldsymbol{N}$ point in Fig. 11, its value (X, ω_c) should correspond to the amplitude and frequency of the self-oscillation appearing in Fig. 3 respectively. ω_c is the cross-frequency of $G(j\omega)$. From Eq. (15), we have:

$$tg0.5\omega_c + \frac{8.44\omega_c - 0.51\omega_c^3}{1 - 4.99\omega_c^2} = 0. \tag{16}$$

Solving Eq. (16)

$\omega_c = 5.72.$

Substituting $\omega_c = 5.72$ into the characteristic equation $1 + N_{RED}(X)G(j\omega) = 0$ and rearranging yields:

$$\frac{0.2}{\pi}\left(\sin^{-1}\frac{150}{X} - \sin^{-1}\frac{75}{X} - \frac{75}{X}\sqrt{1-\left(\frac{75}{X}\right)^2}\right) + \frac{300}{\pi X}\sqrt{1-\left(\frac{150}{X}\right)^2} - 0.056 = 0. \tag{17}$$

Solving Eq. (17)

$X = 151.$

Then, the corresponding amplitude and period of limit cycle are 151 packets and 1.01 s. The values calculated with the describing function approach approximately agree with the results obtained through observing Fig. 3. This shows that it is reasonable to analyze TCP/RED system using the describing function approach.

5.3. Impact of link capacity

The previous works revealed the TCP subsystem, including link capacity C, the number of connections N, and round-trip time (RTT) R, severely affect the stability [8,9]. Subsequently, applying our proposition, we study how these typical network configuration parameters influence the stability of TCP/RED dynamics one by one. For the convenience of analysis and discussion, *Firstly, assume that* given a common network configuration, namely $N = 60$, $R = 0.075$ s, $L = 100$ packets, $H = 150$ packets, $B = 300$ packets, $\max_p = 0.1$, and $w_q = 0.002$, $C = 15$ Mbps. Firstly, assume that *vary* the link capacity C varies from 1 Mbps to 30 Mbps, calculate $G_0(j\omega_c)$ and the maximum value of $-1/N_{0red}(X)$, and plot the results in Fig. 12. After $\boldsymbol{C}$ exceeds 15 Mbps, $\max(-1/N_{0red}(X))$ is more than $G_0(j\omega_c)$, which means that the system must occur a limit cycle. In order to verify this prediction, we construct the simulation network configuration defined by the given parameters, and then repeat the experiment with $C = 10$ Mbps, 15 Mbps, and 20 Mbps. The results presented in Fig. 13 agree with our analysis on the whole.

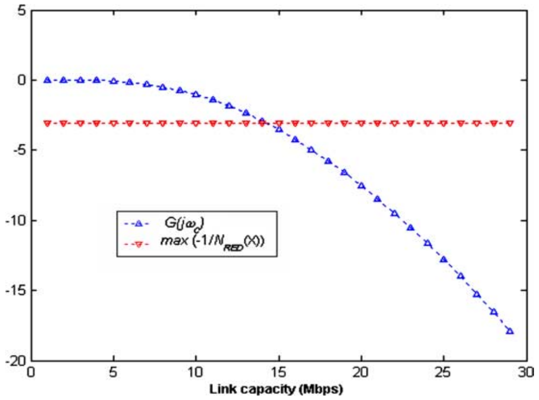

Fig. 12. $G_0(j\omega_c)$ and $\max(-1/N_{0red}(X))$ for different $\boldsymbol{C}$.

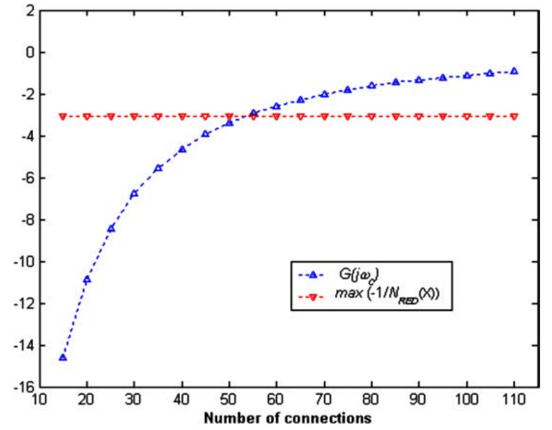

Fig. 14. $G_0(j\omega_c)$ and $\max(-1/N_{0red}(X))$ for different $\boldsymbol{N}$.

5.4. Impact of the number of connections

Fix the link capacity at 15 Mbps, keep other parameters unchangeable except for increasing the number of connections from 10 to 120, and calculate $G_0(j\omega_c)$ and $\max(-1/N_{0red}(X))$ according to the definition in our proposition. The results are depicted in Fig. 14. $\max(-1/N_{0red}(X))$ is an increasing function of the number of connections. The small number of connections readily leads system oscillation. The larger the number of connections is, the more stable the TCP/RED system is. Observing Fig. 14, 55 is the critical number of connection. Choose 40, 60 and 100 connections to do simulations respectively. The simulation results demonstrated in Fig. 15 validate our analysis results.

5.5. Impact of RTT

The delay always takes the negative impact on the system stability in the viewpoint of control theory. We fix the link capacity at 15 Mpbs (3750 packets/s) and the number of connections at 60, and vary the propagation delay, so that RTT is increased from 0.02 s to 0.2 s. $G_0(j\omega_c)$ and $\max(-1/N_{0red}(X))$ are computed in the same way. The results plotted in Fig. 16 show that

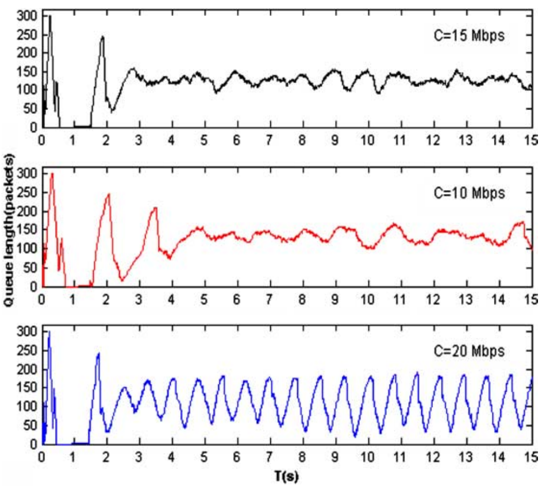

Fig. 13. Queue evolution for different $\boldsymbol{C}$.

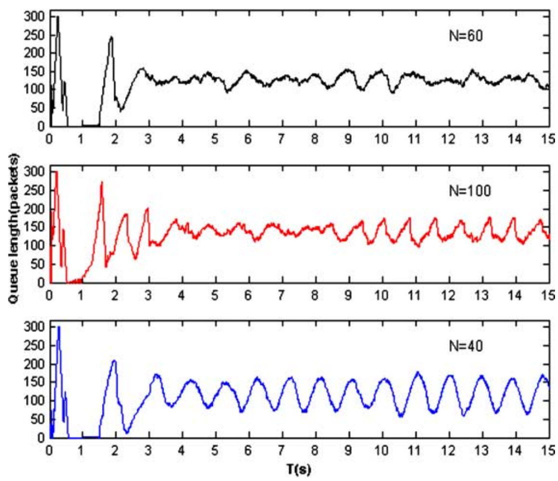

Fig. 15. Queue evolution for different $\boldsymbol{N}$.

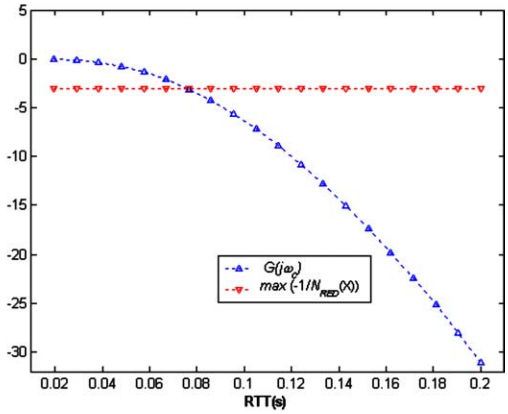

Fig. 16. $G_0(j\omega_c)$ and $\max(-1/N_{0\mathrm{red}}(X))$ for different RTT.

$\max(-1/N_{0\mathrm{red}}(X))$ drastically decreases when RTT increases. We choose 50 ms, 75 ms and 100 ms as typical RTT values to configure simulation, and do the experiments. The queue evolutions are depicted in Fig. 17 respectively. As prediction, the queue is stable when RTT equals 50 ms or 75 ms, whereas, in the network with 100 ms RTT, the queue must be unstable, because the critical value approximately equals to 80 ms. In addition to, when RTT approaches the critical stability value, the queue begins to appear some great fluctuations, although it is still stable and convergent. Comparing the queue evolution whose RTT is 50 ms with that of 75 ms, this phenomena is obvious.

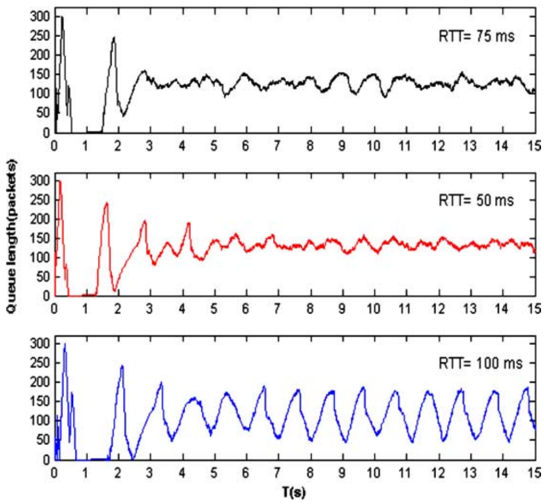

Fig. 17. Queue evolution for different RTT.

5.6. Impact of weight factor

It is difficult job to tune RED for stable operation. The original paper RED [4] presented the impact of parameters on performance, and suggested the guidelines towards the appropriate values. Subsequently, further investigation found that they are not optimal when the Internet backbone upgrades from Kpbs to Mbps, and then offered the current suggestions [20], such as, the weighted factor w_q was increased because it is too small to timely detect the congestion in the relatively high-speed network. This modification was based on simulation experiments and intuitions, and did not give sufficient theoretic analysis. It is likely that the current parameter settings would prove inappropriate again when the speed of Internet backbone upgrades to terabit magnitude since the intuition and partial experience are not always scientific and reasonable under all conditions. Next, we discuss the RED parameters configuration applying our proposition. Let $N = 60$, $R = 0.075$ s, $C = 15$ Mbps, $\min_{th} = 100$ packets, $\max_{th} = 150$ packets, $B = 300$ packets, and $\max_p = 0.1$, increase w_q from 0.001 to 0.1, and calculate $G_0(j\omega_c)$. The results are plotted in Fig. 18. $G_0(j\omega_c)$ curve is concave, the minimum value appears at the point where $w_q = 0.003$ nearby. The smaller $G_0(j\omega_c)$, the more possible $G_0(j\omega)$ intersects with $-1/N_{0\mathrm{red}}(X)$. Thus, it is ineffective to blindly increase w_q in order to avoid the potential conges-

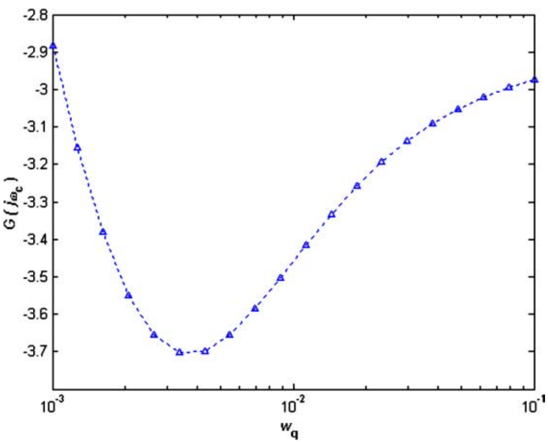

Fig. 18. $G_0(j\omega_c)$ curves with varying w_q.

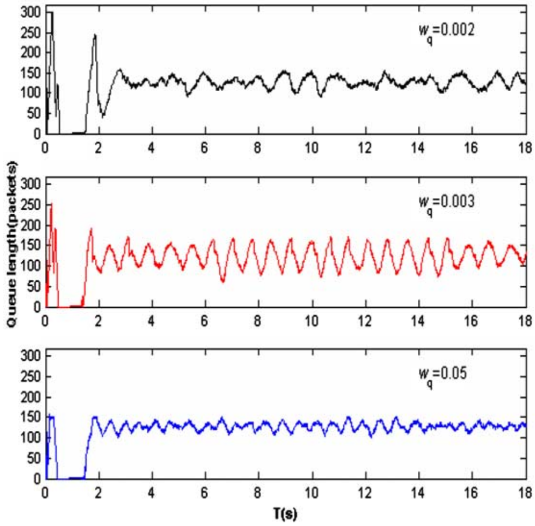

Fig. 19. Queue evolution for different w_q.

tion. On the contrary, it is likely to make a negative impact. The simulation results presented in Fig. 19 verify this argument. The queue is stable when w_q is either 0.002 or 0.05, but it has a limit cycle while w_q is 0.03.

6. Conclusion

RED is an effective AQM implementation algorithm. The investigation about its performance absorbs much attention in the field of current Internet community. Many researches intended to explore the possible reasons why RED algorithm leads the queue oscillation under some conditions, at the same time, some works also provided the parameter settings guidelines for RED stable operation. The existed works can be classified into the two categories in principle. One adopts the continuous-time model, and another follows the discrete-time model. Both of them contribute some rather revelatory conclusions. The former finds that several dominant system parameters affect TCP/RED system operation, and the latter emphasize that the structural nonlinearity existing in algorithm is primary factor leading queue oscillation. However, neither of them can supply a relatively comprehensive explanation why the TCP/RED readily oscillate. To find a more sufficient and reliable reason about TCP/RED instability, a feasible and effective solution should be to combine these two aspects works. We put this idea into practice through applying the describing function approach. After deducing a proposition about TCP/RED stability criterion, we use it to quantitatively study why gentle-RED is more stable than RED, and to discuss the impact of system parameters on TCP/RED system stability in detail. The simulation results validate our analysis and prediction. In addition to, we also clarify some specious viewpoints about parameter configuration.

Acknowledgments

We thank Yin Xunhe for many useful comments and fruitful discussion, and appreciate Prof. Steven Low for his suggestion and encouragement to us. We also thank the anonymous reviewers for their constructive comments. This work is funded by the National Natural Science Foundation of China (No. 60273009), and the Projects of Development Plan of the State Key Fundamental Research in China (No. 2003CB314804).

References

[1] J. Nagle, Congestion control in IP/TCP Internet works, ACM Computer Communication Review 14 (4) (1984) 11–17.

[2] S. Floyd, TCP and explicit congestion notification, ACM Computer Communication Review 24 (October) (1994) 10–23.

[3] B. Braden, D. Clark, J. Crowcroft, et al., Recommendations on queue management and congestion avoidance in the Internet, RFC2309, April 1998.

[4] S. Floyd, V. Jacobson, Random early detection gateways for congestion avoidance, IEEE/ACM Transactions on networking, August 1993.

[5] Martin May, Thomas Bonald, F.D. Smith, Analytic evaluation of RED performance, in: Proc. of IEEE INFOCOM'00, Tel Aviv, Israel, March 2000.

[6] M. Christiansen, K. Jeffay, D. Ott, F.D. Smith, Tuning RED for web traffic, in: Proc. of ACM SIGCOMM 2000, August 2000.

[7] M. May, J. Bolot, C. Diot, B. Lyles, Reasons not to deploy RED, in: Proc. of IWQoS'9, 1997.

[8] C. Hollot, V. Misra, D. Towsley, W.B. Gong, A control theoretic analysis of RED, in: Proc. of IEEE INFOCOM'01, Anchorage, Alaska, 2001.

[9] S.H. Low, F. Paganini, J. Wang, S. Adlakha, J.C. Doyle, Dynamics of TCP/RED and a scalable control, in: Proc. of IEEE INFOCOM'02, New York, USA, 2002.
[10] C.V. Hollot, Y. Chait, Nonlinear stability analysis for a class of TCP/AQM networks, in: IEEE Conference on Decision and Control, 2001, Orlando, pp. 2309–2314.
[11] J. Aweya, M. Ouellette, D.Y. Montuon, A linear system analysis of RED, Computer Communications 25 (2002) 1736–1750.
[12] H. Ohsaki, M. Murata, Steady state analysis of RED gateway: Stability, transient behavior, and parameters setting, IEICE Transactions on Communications E85-B (1) (2002) 107–117.
[13] V. Firoiu, M. Borden. A study of active queue management for congestion control, in: Proc. of IEEE INFOCOM'00, Tel Aviv, Israel, March 2000.
[14] P. Ranjan, E.H. Abed, R.J. La, Nonlinear instabilities in TCP-RED, in: Proc. of IEEE INFOCOM'02, New York, USA, 2002.
[15] V. Misra, W.-B. Gong, D. Towsley, Fluid-based analysis of a network of AQM routers supporting TCP flows with an application to RED, in: Proc. of ACM/SIGCOMM, 2000.
[16] S. Floyd, K. Fall, Promoting the use of end-to-end congestion control in the Internet, IEEE/ACM Transactions on Networking 7 (4) (1999) 458–472.
[17] G.C. Goodwin, S.F. Graebe, M.E. Salgado, Control System Design, Prentice Hall, 2001.
[18] B. Friedland, Advanced Control System Design, Prentice Hall, Englewood Cliffs, 1996.
[19] UCN/LBL/VINT. Network Simulator-NS2, Available from: <http://www-mash.cs.berkeley.edu/ns>.
[20] S. Floyd, RED: Discussion of setting parameters, Available from: <http://www.aciri.org/floyd/redparameters.txt>.

Fengyuan Ren received B.A. and M.Sc. degrees in Automatic Control from Northwestern Polytechnical University, Xi'an, China, in 1993 and 1996 respectively. He obtained Ph.D. degree in Computer Science. During 2000–2001 he worked as a Post Doctor at Electronic Engineering Department of Tsinghua University, Beijing, China. Now, he is an Associate Professor at Computer Science Department of Tsinghua University. His main research interests are Congestion and flow control in computer networks, Control theory application in network systems, and wireless sensor network etc.

Chuang Lin is a full professor of the Department of Computer Science and Technology, Tsinghua University, Beijing, China. He is also a Professor in the Computer Network Information Center at Chinese Academy of Sciences, and in the Information Engineering School at the University of Science and Technology Beijing. He received the Ph.D. degree in Computer Science from Tsinghua University in 1994. In 1985–1986, he was a Visiting Scholar with the Department of Computer Sciences, Purdue University. In 1989–1990, he was a Visiting Research Fellow with the Department of Management Sciences and Information Systems, Graduate School of Business, University of Texas at Austin. In 1995–1996, he visited the Department of Computer Science, Hong Kong University of Science and Technology. His current research interests include computer networks, performance evaluation, logic reasoning, and Petri net theory together with its applications. He has co-authored more than 150 papers in research journals and conference proceedings in these areas.

Bo Wei was born in China on February 3, 1977. He received the B.S. degree in Electrical Engineering from Hebei Institute of Architectural Science and Technology, Handan, China, in 1997 and the M.S. degree from Southern Methodist University in 2001, where he is currently a Ph.D. candidate. His current research interests are in voice compression and transmission over networks.